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The effects of drained and undrained loading on visco-elastic waves in rock-like composites ☆

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Abstract

We outline a micromechanical approach to the acousto-elastic effect in rock-like composites characterized by multiple solid constituents and non-dilute concentrations of interconnected pores and cracks.

Estimates of the T-matrix type (known from the theory of stochastic waves) are first transformed from the stiffness to the compliance domain, so that the (small) strain variation within a single (but interacting) inclusion can be related to the (small) applied effective stress (rather than strain) variation, via a 'non-dilute' K-tensor that depends upon the overall properties as well as the (arbitrary) homogeneous reference material and inclusion (particle, cavity) shape/orientation.

In order to deal with large changes in the applied effective stress under dry conditions, one generally has to integrate a system of ordinary differential equations (ODE's) for the evolution of the microstructural variables (crack densities, porosities, mineral concentrations, aspect ratios of inclusions and correlation functions) under loading.

Under undrained conditions, the (total fluid mass within a representative volume element is conserved) solution to the single cavity deformation problem can be found from the same system of ODE's (and initial conditions) as in the dry case, provided that one replaces dry with saturated (effective compliances) K-tensors depending on second-rank tensors of pore pressure build-up coefficients that can be found from the boundary conditions, in combination with a higher-order expression for the change in porosity (for each cavity type) and the constitutive relation for the fluid.

Under drained conditions, the (fluid pressure is constant) dry system of ODE's can still be used, provided that one replaces (dry with saturated effective compliances) effective with aparent stress variations that depend on the boundary conditions as well as (small) changes in the dry responses during loading. This use of a fluid inclusion-dependent aparent stress in the dry evolution law is possible since the integrated results are independent of the loading-path, in the absence of hysteresis.

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An interacting cavity model of wave-induced fluid flow (which is consistent with the quasi-static considerations outlined above, as well as the Brown–Korringa relations) will finally be presented, and used to estimate the effects of undrained and drained hydrostatic loading on the velocity and attenuation spectra of an isotropic reservoir (example), involving (highly compliant) grain-boundary cracks as well as relatively flat clay-related pores, and more rounded quartz-related pores.

Keywords: Seismic monitoring, Production effects; Sandstone reservoirs; Structural clay; Microscopic cracks

1. Introduction

Attempts to predict changes in the nature and content of pores and cracks in subsurface reservoirs from repeated seismic measurements during 'stressed' times of production have so far been hampered by our inadequate knowledge of real media (e.g., Carcione, 2001), though some promising results have been obtained on the basis of shear-wave splitting (e.g., Angerer et al., 2002) and simple reflection/substitution analysis (e.g., Landrø, 2001). When dealing with natural many-body systems, it clearly makes sense to focus on fractures or cracks (e.g., Zatsepin and Crampin, 1997; Hudson, 2000; Tod, 2002) since even an infinitesimally small concentration of these flat cavities (that respond so dramatically to moderately large changes in the fluid pressure and/or tectonic stresses) may significantly affect the effective stiffness/compliance and permeability tensors. At the same time, it is becoming increasingly clear that one should be careful in (ignoring correlation functions) treating the individual cracks in isolation from each other and from the other inclusions (pores, minerals, etc.) making up a real medium (e.g., Jakobsen et al., 2003a,b; Jakobsen and Hudson, 2003; Jakobsen, 2004a,b).

With applications within time-lapse (4D) seismics in mind, we consider here a new model for the effect of drained and undrained loading on visco-elastic waves in rock-like composites. The terms 'drained' and 'undrained' refer to boundary conditions of constant pore fluid pressure and mass, respectively (see Hudson, 2000). 'Visco-elastic waves' are associated with the phenomenon of wave-induced fluid flow (Hudson et al., 1996; Jakobsen et al., 2003b; Jakobsen, 2004b). 'Rock-like composites' are normally characterized by multiple solid constituents and non-dilute volume concentrations of interconnected pores and cracks (e.g., Thomsen, 1985, 1995; Klimentos and McCann, 1990).

The present study [as well as that of Jakobsen (2004b)] is motivated by the fact that the non-solid inclusions within a rock-like composites can interact in two ways: (1) stress interaction (or strain propagation) associated with Green's function (Hudson, 1980; Ponte Castaneda and Willis, 1995; Jakobsen et al., 2003a; Jakobsen and Hudson, 2003) and (2) fluid pressure communication (or wave-induced fluid flow) associated with stochastic network models (Hudson et al., 1996; Chapman, 2003; Jakobsen et al., 2003b). The effect of stress interaction becomes increasingly important with increasing volume concentrations, and it is rather sensitive to the spatial distribution of inclusions (Ponte Castaneda and Willis, 1995; Jakobsen, 2004a). The effects of pore fluid pressure communication depend on the frequency, but a general theory for this phenomenon should be consistent with the zero-frequency relations of Brown and Korringa (1975) for the effect of fluid-substitution (Thomsen, 1985, 1995).

Published theories of the overall properties of rock-like composites (including cracked media) can be divided into five groups: (1) first-order stiffness-based approximations (Hudson, 1980, 1981; Hudson et al., 1996; Pointer et al., 2000; Chapman, 2003), (2) second-order stiffness-based approximations (based on the method of smoothing) (Hudson, 1980; Hudson et al., 1996), (3) higher-order stiffness-based approximations (based on the variational or T-matrix methods) (Ponte Castaneda and Willis, 1995; Jakobsen et al., 2003a,b; Jakobsen and Hudson, 2003), (4) first-order compliance-based approximations (Sayers and Kahanov, 1995; Schoenberg and Sayers, 1995; Liu et al., 2000) and (5) higher-order compliance-based approximations (Jakobsen, 2004b). The first-order compliance-based approximations contain terms of all order in the volume concentrations of inclusions, if they are expanded as an infinite series of increasing inclusion concentrations. The same statement holds for the higher-order stiffness-based approximations, which take into account the effect of spatial distribution on the basis of two-point statistics.

The first-order stiffness-based approximation is formally valid for crack densities (equant porosities) smaller than 0.1 (10%), whilst real rocks of interest to the petroleum industry frequently have crack densities (equant porosities) equal to 0.3 (40%) or even higher (Thomsen, 1985). The second-order stiffness approximation is known for its prediction of an unphysical increase of effective stiffnessses with increasing crack density, for crack densities larger than around 0.1 (Jakobsen et al., 2003a). The higher-order stiffness-based approximations of Jakobsen et al. (2003a,b) appears to work satisfactory at higher crack densities and equant porosities, but it has not been shown that they do not violate the Brown and Korringa (1975) relations when the spatial distribution of inclusions are no longer the same for all pairs of interacting inclusions. Therefore, Jakobsen (2004b) developed a compliance-based interacting inclusion-based model of wave-induced fluid flow which always has the correct dependency on the saturating fluid (independent of the spatial distribution of inclusions).

Hudson (2000) developed a theory of drained and undrained loading on visco-elastic waves in a special class of rock-like composites involving a single solid constituent and dilute volume concentrations of interconnected cracks (and, to some degree, pores) which is compatible with the first-order stiffness-based approximation of Hudson et al. (1996). [This theory was subsequently implemented for various distributions of crack aspect ratio and orientation by Tod (2002).] The relationship between the present paper and that of Jakobsen (2004b) is analogous to that between the papers of Hudson (2000) and Hudson et al. (1996). The main difference is that we have not followed Hudson et al. in assuming that the response of each individual inclusion is independent of the other inclusions making up the rock-like composite. Since the present theory (but not that of Hudson et al.) takes into account the non-dilute effects of spatial distribution, one could for example use it to model fractures as clusters of cracks (Liu et al., 2000; Jakobsen, 2004a). For many reasons, therefore, it is clear that we have developed a more general theory for the acousto-elastic effect in rock-like composites.

The outline of this paper is simple. We first provide an explicit formula of the T-matrix type for the effective stiffness tensor of a rock-like composite (see Jakobsen et al., 2003a; Jakobsen and Hudson, 2003). We then transform the formula from the stiffness to the compliance domain so that we can estimate the effects of an applied effective stress (rather than strain). After this, we discuss the evaluation of the microstructure under loading. Results are then given for quasi-static loading under undrained and drained loading. Finally, we discuss the anelastic behaviour of seismo-acoustic waves in these structures, on the basis of the (compliance-based) interacting inclusion model of wave-induced fluid flow developed by Jakobsen (2004b). The T-matrix approximations for the effective stiffness tensor given in the next section is needed in order to calculate certain tensors (associated with the responses of the dry inclusions), in the quasi-static cases of drained and undrained loading as well as the dynamic case of wave-induced fluid flow.

2. Inclusion-based models of rocks

We consider here a general class of rock-like composites with non-dilute concentrations of inclusions, that are divided into families of inclusions having the same shape/orientation, t-matrices $\mathbf{t}^{(n)}$ (defined below in terms of stiffness fluctuations), and volume concentration $v^{(n)}$, labelled by $n = 1,2,\dots,N$. In the frequency domain [discussed by Carcione (2001)], we may relate the second-rank tensors of (small) stress ($\delta\langle\boldsymbol{\sigma}\rangle$) and strain ($\delta\langle\boldsymbol{\epsilon}\rangle$) variations by the linear transformation (called Hooke's law in the elastic case)

$$\delta\langle\boldsymbol{\sigma}\rangle = \mathbf{C}^{*} : \delta\langle\boldsymbol{\epsilon}\rangle, \tag{1}$$

where the ':' symbol refers to the double dot product. Here, $\mathbf{C}^*$ is a fourth-rank tensor of (tangential or instantaneous) effective stiffnesses that can be evaluated on the basis of the following T-matrix approximation (Jakobsen et al., 2003a; Jakobsen and Hudson, 2003):

$$\mathbf{C}^* = \mathbf{C}^{(0)} + \mathbf{C}_1 : (\mathbf{I}_4 + \mathbf{C}_1^{-1} : \mathbf{C}_2)^{-1}, \tag{2}$$

$$\mathbf{C}_1 = \sum_{r=1}^{N} v^{(r)} \mathbf{t}^{(r)}, \tag{3}$$

$$\mathbf{C}_2 = \sum_{r=1}^{N} \sum_{s=1}^{N} v^{(r)} \mathbf{t}^{(r)} : \mathbf{G}_{\mathrm{d}}^{(rs)} : \mathbf{t}^{(s)} v^{(s)}. \tag{4}$$

Here $\mathbf{C}^{(0)}$ is the stiffness tensor for a homogeneous reference medium which can be (anisotropic) selected rather arbitrary without violating the mechanical stability criterion; $\mathbf{I}_4$ is the identity for fourth-rank tensors; $\mathbf{G}_{\mathrm{d}}^{(rs)}$ is given by (Appendix A) the strain Green's function (associated with $\mathbf{C}^{(0)}$) integrated over a characteristic ellipsoid having the same symmetries as $p^{(s|r)}(\mathbf{x} - \mathbf{x}')$ which, in turn, gives the probability density for finding an inclusion of type s at point $\mathbf{x}'$ given that there is an inclusion of type r at point $\mathbf{x}$.

The t-matrix for a single inclusion of type r is given by (Jakobsen et al., 2003a)

$$\mathbf{t}^{(r)} = (\mathbf{C}^{(r)} - \mathbf{C}^{(0)}) : [\mathbf{I}_4 - \mathbf{G}^{(r)} : (\mathbf{C}^{(r)} - \mathbf{C}^{(0)})]^{-1}, \tag{5}$$

where $\mathbf{G}^{(r)}$ is a fourth-rank tensor (discussed in Appendix A) depending only on $\mathbf{C}^{(0)}$ and the shape/orientation of the rth inclusion type. In the case of a dry cavity, we may formally set $\mathbf{C}^{(r)} = \mathbf{0}$. In the case of a fully saturated cavity which is not isolated with respect to (wave-induced) fluid flow, the $\mathbf{C}^{(r)}$ tensor represents complex-valued functions of frequency (see Jakobsen et al., 2003b; Jakobsen, 2004b).

3. The response of an interacting inclusion

The T-matrix approximations described above can be rewritten exactly (as suggested by Kailasam et al., 1997)

$$\mathbf{C}^* = \sum_{r=0}^{N} v^{(r)} \mathbf{C}^{(r)} : \mathbf{A}_*^{(r)}, \tag{6}$$

where the 'non-dilute' (or interacting) strain concentration tensors of Jakobsen (2004b);

$$\mathbf{A}_*^{(r)} = \mathbf{A}^{(r)} : [\mathbf{I}_4 + \mathbf{C}_1^{-1} : \mathbf{C}_2]^{-1}, \tag{7}$$

which are related to the 'dilute' (or non-interacting) strain concentration tensors of Eshelby (1957);

$$\mathbf{A}^{(r)} = [\mathbf{I}_4 - \mathbf{G}^{(r)} : (\mathbf{C}^{(r)} - \mathbf{C}^{(0)}]^{-1}, \tag{8}$$

are such that the strain variation within phase r is related to the applied average strain variation by

$$\delta\boldsymbol{\epsilon}^{(r)} = \mathbf{A}_*^{(r)} : \delta\langle\boldsymbol{\epsilon}\rangle; \tag{9}$$

and $\mathbf{A}^{(0)}$ is such that

$$\sum_{r=0}^{N} v^{(r)} \mathbf{A}_*^{(r)} = \mathbf{I}_4. \tag{10}$$

The corresponding estimates for the effective compliance tensor

$$\mathbf{S}^* = (\mathbf{C}^*)^{-1}, \tag{11}$$

are given exactly (as shown by Hudson, 1991)

$$\mathbf{S}^* = \mathbf{S}^{(0)} - \sum_{r=1}^{N} v^{(r)}(\mathbf{S}^{(0)} : \mathbf{C}^{(r)} - \mathbf{I}_4) : \mathbf{K}_*^{(r)}, \tag{12}$$

where the 'non-dilute' K-tensors

$$\mathbf{K}_*^{(r)} = \mathbf{A}_*^{(r)} : \mathbf{S}^*, \tag{13}$$

found by combining Eq. (9) with the effective (tangential) stress–strain relation (1), are such that the strain variation within phase r is related to the applied average stress variation by

$$\delta\boldsymbol{\epsilon}^{(r)} = \mathbf{K}_*^{(r)} : \delta\langle\boldsymbol{\sigma}\rangle. \tag{14}$$

Eqs. (7) and (13) imply that

$$\mathbf{K}_*^{(r)} = \mathbf{K}^{(r)} + \mathrm{O}[v^2], \tag{15}$$

where v denotes $v^{(r)}$, $r = 1,\ldots,N$; and

$$\mathbf{K}^{(r)} = \mathbf{A}^{(r)} : \mathbf{S}^{(0)} \tag{16}$$

is the 'dilute' K-tensor of Eshelby (1957).

4. Evolving microstructures under loading

When a real medium is subjected to a finite deformation, simplifying assumptions are obviously required in order to capture the essential features of the complex microstructure under evolution. The first such assumption will be that the applied effective stress is triaxial with axes coinciding with the symmetry axes of the generally anisotropic medium. This ensures that (on the average at least) the inclusions do not change orientation during the deformation process (see Kailasam et al., 1997). The next assumption will be that the inclusions are ellipsoidal in shape so that they deform into ellipsoids when they are subjected to uniform loading conditions (Eshelby, 1957), (approximately) even in the presence of non-dilute inclusion concentrations (see Zaidman and Ponte Castaneda, 1996; Kailasam et al., 1997). Finally, it will be assumed that the evolution of the aspect ratios of the ellipsoidal correlation functions is determined by the average strain variations in the heterogeneous material as a whole (see Kailasam et al., 1997).

Having identified the most essential microstructural variables, we next write down the corresponding evolution laws [for the special case of fully aligned spheroidal inclusions (with aspect ratios $\alpha^{(r)}$; $r = 1,2,\ldots,N$) that are distributed in space in accordance with two-point correlation functions (having symmetries characterized by a set of spheroids with aspect ratios $\alpha_{\mathrm{d}}^{(rs)}$; $r,s = 1,2,\ldots,N$)]:

$$\frac{\delta v^{(r)}}{v^{(r)}} = \delta\epsilon_{kk}^{(r)} - \sum_{s=1}^{N} v^{(s)}\delta\epsilon_{kk}^{(s)}, \tag{17}$$

$$\frac{\delta\alpha^{(r)}}{\alpha^{(r)}} = \delta\epsilon_{33}^{(r)} - \delta\epsilon_{11}^{(r)}, \tag{18}$$

$$\frac{\delta\alpha_{\mathrm{d}}^{(rs)}}{\alpha_{\mathrm{d}}^{(rs)}} = \delta\langle\epsilon_{33}\rangle - \delta\langle\epsilon_{11}\rangle. \tag{19}$$

The second term on the right-hand side of Eq. (17) (derived in Appendix B) can be ignored if and only if the inclusion concentrations are dilute, as discussed (in connection with a less general expression) by Kailasam et al. (1997). Eqs. (18) and (19) were also discussed by Kailasam et al. (1997), though these authors assumed the spatial distribution to be the same for all pairs of interacting inclusions, in the sense that the aspect ratios [of the 'ellipsoidal' correlation functions, $p^{(s|r)}(\mathbf{x}-\mathbf{x}')$] $\alpha_{\mathrm{d}}^{(rs)}=\alpha_{\mathrm{d}}$ for all r and s.

If we for simplicity write

$$\delta\langle\boldsymbol{\sigma}\rangle=\delta P\hat{\boldsymbol{\sigma}}, \tag{20}$$

where (δP denotes the magnitude of the small variation in the applied effective stress) the components of the second-rank tensor $\hat{\boldsymbol{\sigma}}$ are given by $\hat{\sigma}_{pq}=-\delta_{pq}$ for a hydrostatic stress, $\hat{\sigma}_{pq}=-\delta_{p3}\delta_{q3}$ for a uniaxial stress along the x_3-axis, then we obtain from the above evolution laws [in conjunction with Eqs. (1) and (13)] a system of ODE's;

$$\frac{\mathrm{d}v^{(r)}}{\mathrm{d}P}=v^{(r)}\left[\left(K_*^{(r)}\right)_{kkpq}-\sum_s v^{(s)}\left(K_*^{(s)}\right)_{kkpq}\right]\hat{\sigma}_{pq}, \tag{21}$$

$$\frac{\mathrm{d}\alpha^{(r)}}{\mathrm{d}P}=\alpha^{(r)}\left[\left(K_*^{(r)}\right)_{33pq}-\left(K_*^{(r)}\right)_{11pq}\right]\hat{\sigma}_{pq}, \tag{22}$$

$$\frac{\mathrm{d}\alpha_{\mathrm{d}}^{(rs)}}{\mathrm{d}P}=\alpha_{\mathrm{d}}^{(rs)}[S^*_{33pq}-S^*_{11pq}]\hat{\sigma}_{pq}, \tag{23}$$

that can be solved by using the Runge–Kutta method.

5. Quasi-static loading under undrained conditions

By linear superposition, the strain variation $\delta\epsilon^{(r)}$ within a cavity of type r that is completely saturated with fluid under the pressure variation δp_{f}, when subjected to a (small) variation $\delta\langle\boldsymbol{\sigma}\rangle$ in the effective stress, is given by (Jakobsen, 2004b)

$$\mathbf{K}_*^{(r)}:\delta\langle\boldsymbol{\sigma}\rangle=\mathbf{K}_{\mathrm{d}*}^{(r)}:(\delta\langle\boldsymbol{\sigma}\rangle+\mathbf{I}_2\delta p_{\mathrm{f}})-\mathbf{S}^{(0)}:\mathbf{I}_2\delta p_{\mathrm{f}}, \tag{24}$$

were $\mathbf{K}_{d*}^{(r)}$ obviously refers to the dry response and $\mathbf{I}_2$ is the identity for second-rank tensors. The above superposition is useful because $\mathbf{K}_{\mathrm{d}*}^{(r)}$ can easily be determined from the following set of equations:

$$\mathbf{K}_{\mathrm{d}*}^{(r)}=\mathbf{A}_{\mathrm{d}*}^{(r)}:\mathbf{S}_{\mathrm{d}}^*, \tag{25}$$

$$\mathbf{S}_{\mathrm{d}}^*=(\mathbf{C}_{\mathrm{d}}^*)^{-1}, \tag{26}$$

$$\mathbf{C}_{\mathrm{d}}^*=\mathbf{C}^{(0)}+\mathbf{C}_{1\mathrm{d}}:(\mathbf{I}_4+\mathbf{C}_{1\mathrm{d}}^{-1}:\mathbf{C}_{2\mathrm{d}})^{-1}, \tag{27}$$

$$\mathbf{C}_{1\mathrm{d}}=\sum_{r=1}^{N_{\mathrm{c}}}v^{(r)}\mathbf{t}_{\mathrm{d}}^{(r)}+\sum_{r=N_{\mathrm{c}}+1}^{N}v^{(r)}\mathbf{t}^{(r)}, \tag{28}$$

$$\begin{aligned}\mathbf{C}_{2\mathrm{d}}=&\sum_{r=1}^{N_{\mathrm{c}}}\sum_{s=1}^{N_{\mathrm{c}}}v^{(r)}\mathbf{t}_{\mathrm{d}}^{(r)}:\mathbf{G}_{\mathrm{d}}^{(rs)}:\mathbf{t}_{\mathrm{d}}^{(s)}v^{(s)}+\sum_{r=1}^{N_{\mathrm{c}}}\sum_{s=N_{\mathrm{c}}+1}^{N}v^{(r)}\mathbf{t}_{\mathrm{d}}^{(r)}:\mathbf{G}_{\mathrm{d}}^{(rs)}:\mathbf{t}^{(s)}v^{(s)}+\sum_{r=N_{\mathrm{c}}+1}^{N}\sum_{s=1}^{N_{\mathrm{c}}}v^{(r)}\mathbf{t}^{(r)}:\mathbf{G}_{\mathrm{d}}^{(rs)}:\mathbf{t}_{\mathrm{d}}^{(s)}v^{(s)}\\&+\sum_{r=N_{\mathrm{c}}+1}^{N}\sum_{s=N_{\mathrm{c}}+1}^{N}v^{(r)}\mathbf{t}^{(r)}:\mathbf{G}_{\mathrm{d}}^{(rs)}:\mathbf{t}^{(s)}v^{(s)},\end{aligned} \tag{29}$$

$$\mathbf{t}_{\mathrm{d}}^{(r)} = -\mathbf{C}^{(0)} : [\mathbf{I}_4 + \mathbf{G}^{(r)} : \mathbf{C}^{(0)}]^{-1}, \tag{30}$$

$$\mathbf{A}_{\mathrm{d}*}^{(r)} = \mathbf{A}_{\mathrm{d}}^{(r)} : [\mathbf{I}_4 + \mathbf{C}_{1\mathrm{d}}^{-1} : \mathbf{C}_{2\mathrm{d}}]^{-1}, \tag{31}$$

$$\mathbf{A}_{\mathrm{d}}^{(r)} = [\mathbf{I}_4 + \mathbf{G}^{(r)} : \mathbf{C}^{(0)}]^{-1}, \tag{32}$$

corresponding with Eqs. (13), (11), (2)–(5), (7), (8), respectively. [A subscript 'd' is used here on all quantities associated with dry cavities, characterized by vanishing stiffnesses.] In the above equations, N_{c} denote the number of cavity types and $(N - N_{\mathrm{c}})$ obviously refers to the number of solid constituents. The first, second, third and fourth term on the right-hand side of Eq. (29) is associated with (dry) cavity–cavity interactions, cavity–solid interactions, solid–cavity interactions, and solid-solid interactions, respectively.

From the linearity of the problem we know that there exists a second-rank tensor **B** of (Skempton's) pore pressure build-up coefficients (see Green and Wang, 1986; Berge et al., 1993), so that

$$\delta p_{\mathrm{f}} = \mathbf{B} : \delta\langle\boldsymbol{\sigma}\rangle. \tag{33}$$

By using Eqs. (24) and (33) in conjunction with the fact that $\delta\langle\boldsymbol{\sigma}\rangle$ is arbitrary, we find that

$$\mathbf{K}_{*}^{(r)} = \mathbf{K}_{\mathrm{d}*}^{(r)} + (\mathbf{K}_{\mathrm{d}*}^{(r)} - \mathbf{S}^{(0)}) : (\mathbf{I}_2 \otimes \mathbf{B}), \tag{34}$$

where the symbol $\otimes$ denotes the dyadic tensor product.

To find **B** in the case of an undrained quasi-static loading, we require that the total fluid mass m_{f} within the whole population of N_{c} (interconnected) cavities is conserved. Thus, if $v^{(r)}/\rho_{\mathrm{f}}$ and $\tilde{v}^{(r)}/\tilde{\rho}_{\mathrm{f}}$ are the unstressed and stressed porosities/densities then

$$m_{\mathrm{f}} = \sum_{r=1}^{N_{\mathrm{c}}} v^{(r)}\rho_{\mathrm{f}} = \sum_{r=1}^{N_{\mathrm{c}}} \tilde{v}^{(r)}\tilde{\rho}_{\mathrm{f}}, \tag{35}$$

where, the constitutive relation for the fluid is,

$$\tilde{\rho}_{\mathrm{f}} = \frac{\rho_{\mathrm{f}}}{1 - \delta p_{\mathrm{f}}/\kappa_{\mathrm{f}}}, \tag{36}$$

and κ_{f} is the fluid bulk modulus. From Eqs. (14) and (24), it follows that

$$\frac{\tilde{v}^{(r)} - v^{(r)}}{v^{(r)}} = (K_{\mathrm{d}*}^{(r)})_{uupq}(\delta\langle\sigma_{pq}\rangle + \delta_{pq}\delta p_{\mathrm{f}}) - S_{uuvv}^{(0)}\delta p_{\mathrm{f}}. \tag{37}$$

Eqs. (33), (35)–(37) imply that

$$\mathbf{B} = -\Theta_{\mathrm{s}*}\mathbf{I}_2 : \sum_{r=1}^{N_{\mathrm{c}}} v^{(r)}\mathbf{K}_{\mathrm{d}*}^{(r)}, \tag{38}$$

$$\Theta_{\mathrm{s}*} = \left[\left(\frac{1}{\kappa_{\mathrm{f}}} - (S^{(0)})_{uuvv}\right)\phi + \sum_{r=1}^{N_{\mathrm{c}}} v^{(r)}(K_{\mathrm{d}*}^{(r)})_{uuvv}\right]^{-1}, \tag{39}$$

where ϕ is the total porosity. Combining Eqs. (34) and (38), we get

$$\mathbf{K}_{*}^{(n)} = \mathbf{K}_{\mathrm{d}*}^{(n)} - \Theta_{\mathrm{s}*}(\mathbf{K}_{\mathrm{d}*}^{(n)} - \mathbf{S}^{(0)}) : (\mathbf{I}_2 \otimes \mathbf{I}_2) : \sum_{r=1}^{N_{\mathrm{c}}} v^{(r)}\mathbf{K}_{\mathrm{d}*}^{(r)}. \tag{40}$$

6. Quasi-static loading under drained conditions

Eq. (24) can be rewritten exactly as

$$\mathbf{K}_{*}^{(r)} : \delta\langle\boldsymbol{\sigma}\rangle = \mathbf{K}_{\mathrm{d}*}^{(r)} : \delta\langle\boldsymbol{\sigma}\rangle_{a}^{(r)}, \tag{41}$$

where

$$\delta\langle\boldsymbol{\sigma}\rangle_{a}^{(r)} = \delta\langle\boldsymbol{\sigma}\rangle + \delta p_{\mathrm{f}}\boldsymbol{\alpha}^{(r)}, \tag{42}$$

and

$$\boldsymbol{\alpha}^{(r)} = [\mathbf{I}_2 - (\mathbf{K}_{\mathrm{d}*}^{(r)})^{-1} : \mathbf{S}^{(0)} : \mathbf{I}_2]. \tag{43}$$

Here, $\boldsymbol{\alpha}^{(r)}$ is a second-rank tensor of apparent stress-coefficients (see Hudson, 2000) that can be used at non-dilute inclusion concentrations, in contrast with the work of Zatsepin and Crampin (1997). Since the integrated results are independent of the loading-path (in the absence of hysteresis), we may use (21) and (22) in connection with the $\mathbf{K}_{\mathrm{d}*}^{(r)}$, provided that $\hat{\boldsymbol{\sigma}} \to \hat{\boldsymbol{\sigma}}_{a}^{(r)} = \hat{\boldsymbol{\sigma}} + (p_{\mathrm{f}}/P)\boldsymbol{\alpha}^{(r)}$, where p_{f} and P are the (finite) values of pore fluid pressure and effective stress we wish to impose.

7. Visco-elastic waves

The propagation of visco-elastic waves in rock-like composites implies a dynamic situation, which must be consistent with the quasi-static considerations of the previous sections, as well as the (anisotropic Gassmann) relations of Brown and Korringa (1975) for the (zero frequency) dependence of the elastic compliances of a porous rock on the compressibility of the pore fluid. If and only if the spatial distributions are taken to be the same for all pairs of interacting inclusions, the standard (stiffness-based) T-matrix approach to wave-induced fluid flow discussed by Jakobsen et al. (2003b) satisfies these criteria. In the revised (complianced-based) theory of Jakobsen (2004b), however, the (criteria are always satisfied) saturated effective compliance tensor $\mathbf{S}^{*}$ is given in terms of the dry result $\mathbf{S}_{\mathrm{d}}^{*}$ by

$$\mathbf{S}^{*} = \mathbf{S}_{\mathrm{d}}^{*} - \Theta_{*}\mathbf{L} : (\mathbf{I}_2 \otimes \mathbf{I}_2) : \mathbf{L} - \mathrm{i}\omega\tau\kappa_{\mathrm{f}}\mathbf{H}, \tag{44}$$

$$\Theta_{*} = \kappa_{\mathrm{f}}\left[(1 - \kappa_{\mathrm{f}}S_{uuvv}^{(0)})\Sigma_a + \kappa_{\mathrm{f}}\Sigma_b - \frac{\mathrm{i}k_u k_v \Gamma_{uv}\kappa_{\mathrm{f}}}{\eta_{\mathrm{f}}\omega}\right]^{-1}, \tag{45}$$

$$\Sigma_a = \sum_{r=1}^{N_{\mathrm{c}}} \frac{v^{(r)}}{1 + \mathrm{i}\omega\gamma_{*}^{(r)}\tau}, \tag{46}$$

$$\Sigma_b = \sum_{r=1}^{N_{\mathrm{c}}} \frac{v^{(r)}(K_{\mathrm{d}*}^{(r)})_{uuvv}}{1 + \mathrm{i}\omega\gamma_{*}^{(r)}\tau}, \tag{47}$$

$$\gamma_{*}^{(r)} = 1 + \kappa_{\mathrm{f}}(K_{\mathrm{d}*}^{(r)} - S^{(0)})_{uuvv}, \tag{48}$$

$$\mathbf{L} = \sum_{r=1}^{N_{\mathrm{c}}} \frac{v^{(r)}}{1 + \mathrm{i}\omega\gamma_{*}^{(r)}\tau}\mathbf{K}_{\mathrm{d}*}^{(r)}, \tag{49}$$

$$\mathbf{H} = \sum_{r=1}^{N_{\mathrm{c}}} \frac{v^{(r)}}{1 + \mathrm{i}\omega\gamma_{*}^{(r)}\tau}\mathbf{K}_{\mathrm{d}*}^{(r)} : (\mathbf{I}_2 \otimes \mathbf{I}_2) : \mathbf{K}_{\mathrm{d}*}^{(r)}. \tag{50}$$

Here Γ_{uv} is a component of the effective permeability tensor; τ is the squirt flow relaxation time constant [discussed by Jakobsen et al. (2003b)]; η_f is the viscosity of the fluid; and k_u is a component of the wave vector. For simplicity, we normally assume that $k_u = \omega s_u$, where ω is the angular frequency and s_u is a component of the slowness vector for plane waves in the homogeneous reference medium.

8. Example

The results of Jakobsen et al. (2003b) suggest that clayey sandstones can safely be treated as visco-elastic composites on the basis of a dual porosity model which is quite similar to that of Xu and White (1995), but (representing the clay phase in the form of isolated inclusions, within a loading–bearing matrix of quartz) taking the phenomenon of wave-induced fluid flow into account. In the (generalized) Xu–White model, the total pore space is assumed to consist of two parts: (1) pores associated with quartz grains and (2) pores associated with clays. The essential feature of the model is the assumption that the clay-related pores are significantly flatter than the quartz-related pores. In this study, we follow Jakobsen et al. (2003b) in taking $\alpha^{(1)} = 0.15$ and $\alpha^{(2)} = 0.027$ for the unstressed rock, where $\alpha^{(1)}$ and $\alpha^{(2)}$ are the aspect ratios of the quartz- and clay-related pores, respectively.

Klimentos and McCann (1990) measured P-wave speeds and attenuations at 1 MHz in a suite of clayey sandstones under (drained conditions) variable confining pressure. At high confining (or differential) pressure, all cracks are probably closed, and this may explain why the uncracked model of Jakobsen et al. (2003b) worked so well. At lower confining (or differential) pressures, however, some of the flattest cavities may still be open, and so we generally need to allow for the existence of cracks in these complex porous media. Jakobsen and Hudson (2003) extended the model of Jakobsen et al. (2003b) to include cracks,

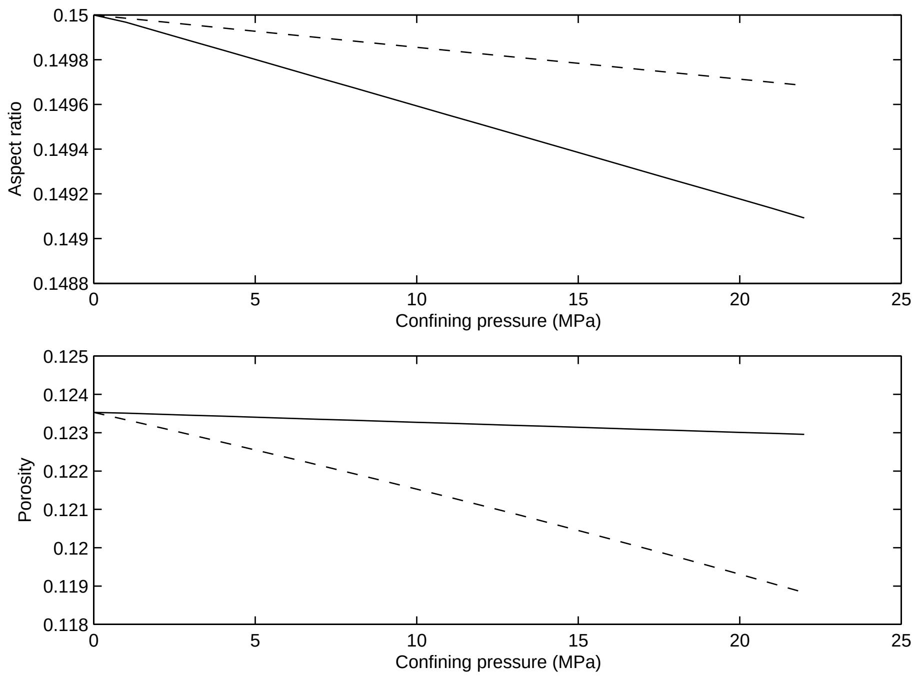

Fig. 1. Quartz-related pores under stress. Solid and dashed curves correspond with drained and undrained boundary conditions, respectively.

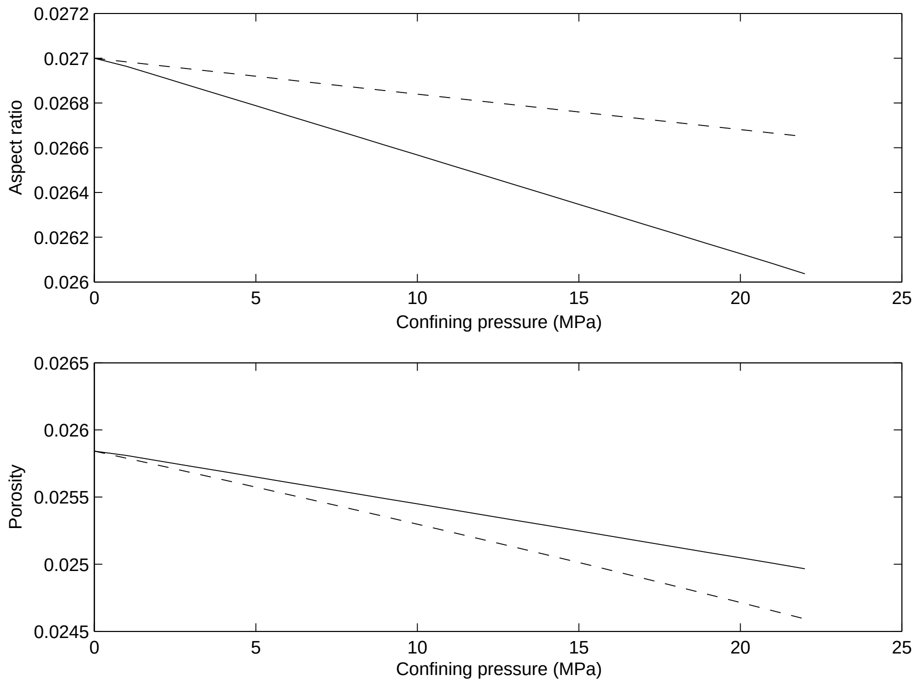

Fig. 2. Clay-related pores under stress. Solid and dashed curves correspond with drained and undrained boundary conditions, respectively.

but studied the effects of crack density separately from those of the crack aspect ratio. In this study, we allowed the aspect ratio to depend upon the crack density via the applied effective stress, under various boundary conditions.

Figs. 1–3 show the aspect ratios and porosities of the various cavities within our clayey sandstone model. Clearly, it is the cracks that represent the most compliant part of the pore space. However, we can also see a significant pressure-induced change in the geometry of the clay-related pores. The quartz-related pores (and even more so, the clay particles) are relatively non-compliant. Fig. 4 shows the pore fluid pressure as a function of confining pressure. In the undrained case, the pore fluid pressure is determined by the applied confining pressure via the **B** tensor in Eq. (33). In the drained case, however, the solid curves merely show what path we have taken in the stress–pressure plane, in order to reach our final destination; that is, a confining pressure of 22 MPa and a pore fluid pressure of 4.4 MPa. The acousto-elastic results (for two different confining pressures) in Fig. 5 suggest that it is very important to take into account the (fluid dynamical) boundary conditions when trying to predict the effects of an applied effective stress on the seismic wave characteristics of real media like dirty sandstone reservoirs. The initial crack density (aspect ratio) was 0.15 (0.001), and the cracks closed completely in the case of drained loading only.

9. Concluding remarks

We have approached the acousto-elastic effect in real media from the viewpoint of a revised T-matrix approach to rock physics, in the sense that the response of a single inclusion (or crack) under an applied effective stress was allowed to be modified by its interaction with other inclusions. We have seen that often takes a significantly smaller stress to close an interacting crack than an isolated one, and the response of a

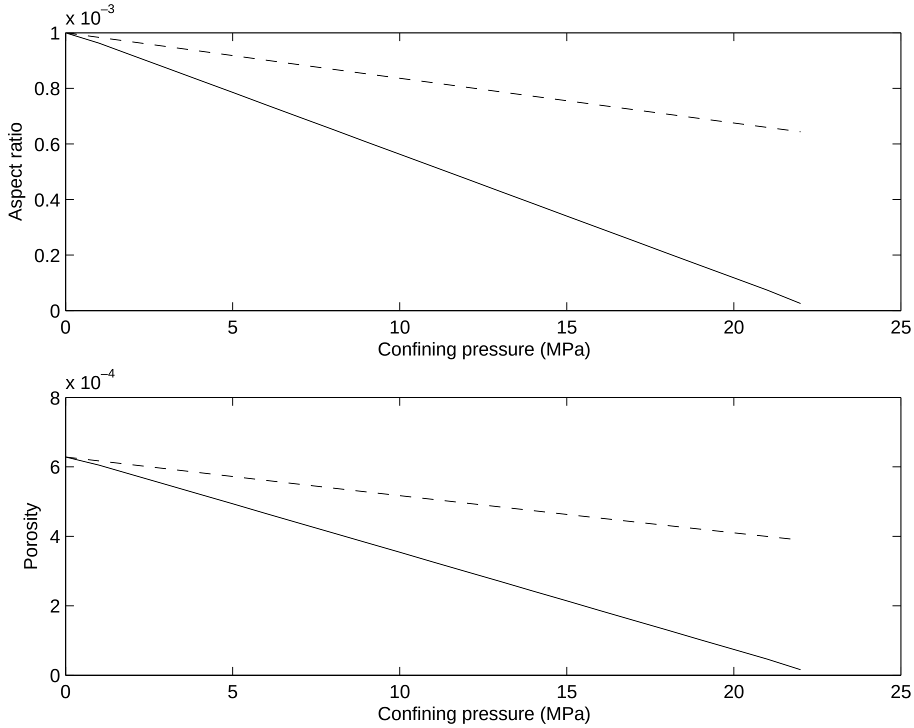

Fig. 3. Cracks under stress. Solid and dashed curves correspond with drained and undrained boundary conditions, respectively.

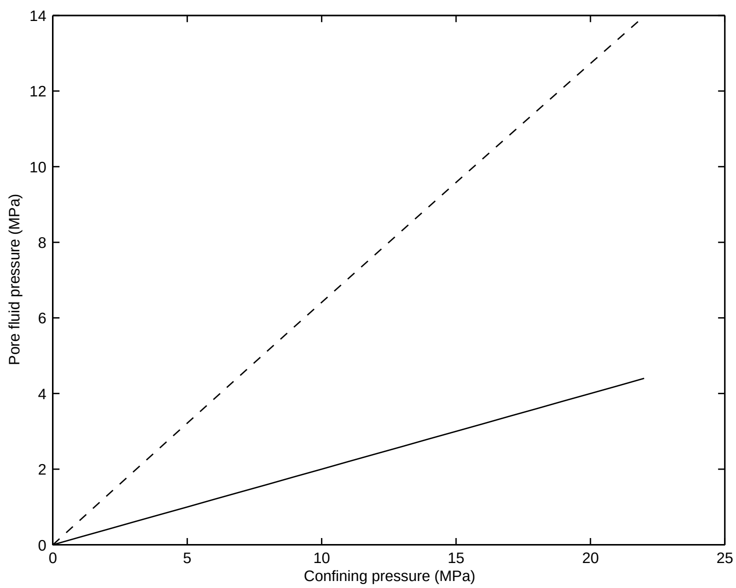

Fig. 4. Pore fluid pressure versus confining pressure. Solid and dashed curves correspond with drained and undrained boundary conditions, respectively.

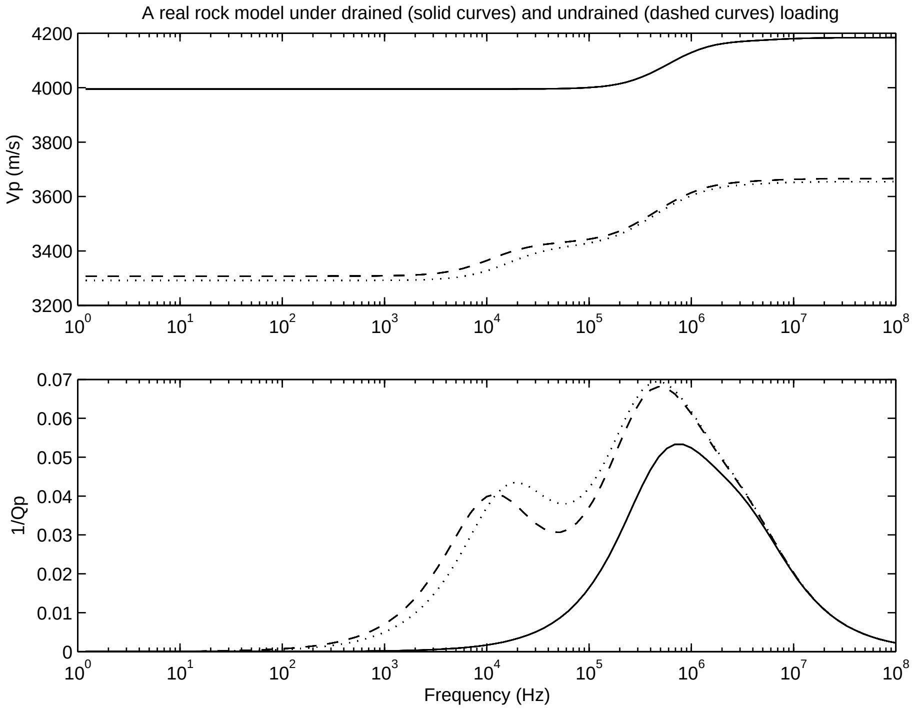

Fig. 5. P-wave characteristics of a dual porosity clay–sand mixture [similar to that of Jakobsen et al. (2003b) but including cracks] under 22 MPa confining pressure, if not unstressed (dotted curves). These results were obtained on the basis of a combination of an effective medium approximation for the frequency-dependent and complex-valued effective stiffness tensor with the plane wave equations of Carcione (2001). As discussed by Jakobsen et al. (2003b), the effect of increasing the crack-size is to increase the value of τ, so that the attenuation peaks move towards the lower frequencies important for seismic reservoir characterization and monitoring. Note that the effect of crack closure (solid curves, drained conditions) is to increase the wave speeds and decrease the attenuations.

single crack is very sensitive to the (drained versus undrained) boundary conditions. The fact that we can now deal the non-linear behaviour of non-dilute inclusion concentrations appears to be rather important, but our expression for Skempton's pore pressure build-up tensor **B** in terms of the parameters of the microstructure may be even more interesting. [The plots in Figs. 1–4 may be (consistent with the work of Hudson, 2000) rather linear, but a non-linear behaviour has indeed been observed in other computations, involving higher values of the applied effective stress.] The theory and code we have developed may for example be used (in the context of time-lapse seismics) to discriminate between large changes in pore pressure and fluid-saturation during production (see Landrø, 2001; Koesoemadinata and McMechan, 2001), or (in the related context of compaction) to simulate the seismic changes of pores that are deformed into cracks (see Ruud et al., 2003). The next step may be to implement the theory for an applied effective stress which is not hydrostatic and/or to consider less trivial distributions of cavity orientations/shapes (see Tod, 2002).

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Appendix A. Evaluation of the tensors $\mathbf{G}^{(r)}$ and $\mathbf{G}_{\mathrm{d}}^{(rs)}$

It is clear from Eq. (8) and the work of Torquato (2002) that the tensor $\mathbf{G}^{(r)}$ is given by

$$\mathbf{G}^{(r)} = -\mathbf{S}^{(r)} : \mathbf{S}^{(0)}, \tag{A.1}$$

where $\mathbf{S}^{(r)}$ is the so-called Eshelby (1957) tensor of the ellipsoid. Eshelby's tensor is generally given in terms of elliptic integrals of the first and second kinds (Mura, 1982).

In the case an isotropic matrix material containing spheroidal inclusions with semiaxes $a_1^{(r)} = a_2^{(r)} = a_r$ and $a_3^{(r)} = b_r$ and whose symmetry axis is aligned in the x_3-direction, the elliptic integrals can be evaluated analytically (Mura, 1982).

If the matrix material is isotropic then the components of $S_{ijkl}^{(r)}$ are given by (Torquato, 2002)

$$\begin{aligned}
S_{1111}^{(r)} = S_{2222}^{(r)} &= \frac{3}{8(1-v)}\frac{\alpha_r^2}{\alpha_r^2-1} + \frac{1}{4(1-v)}\left[1-2v-\frac{9}{4(\alpha_r^2-1)}\right]q,\\
S_{3333}^{(r)} &= \frac{1}{2(1-v)}\left\{1-2v+\frac{3\alpha_r^2-1}{\alpha_r^2-1}-\left[1-2v+\frac{3\alpha_r^2}{\alpha_r^2-1}\right]q\right\},\\
S_{1122}^{(r)} = S_{2211}^{(r)} &= \frac{1}{4(1-v)}\left\{\frac{\alpha_r^2}{2(\alpha^2-1)}-\left[1-2v+\frac{3}{4(\alpha_r^2-1)}\right]q\right\},\\
S_{1133}^{(r)} = S_{2233}^{(r)} &= \frac{1}{2(1-v)}\left\{\frac{-\alpha_r^2}{\alpha_r^2-1}+\frac{1}{2}\left[\frac{3\alpha_r^2}{\alpha_r^2-1}-(1-2v)\right]q\right\},\\
S_{3311}^{(r)} = S_{3322} &= \frac{1}{2(1-v)}\left\{2v-1-\frac{1}{\alpha_r^2-1}+\left[1-2v+\frac{3}{2(\alpha_r^2-1)}\right]q\right\},\\
S_{1212}^{(r)} &= \frac{1}{4(1-v)}\left\{\frac{\alpha_r^2}{2(\alpha_r^2-1)}+\left[1-2v-\frac{3}{4(\alpha_r^2-1)}\right]q\right\},\\
S_{1313}^{(r)} = S_{2323}^{(r)} &= \frac{1}{4(1-v)}\left\{1-2v-\frac{\alpha_r^2+1}{\alpha_r^2-1}-\frac{1}{2}\left[1-2v-\frac{3(\alpha_r^2+1)}{\alpha_r^2-1}\right]q\right\},
\end{aligned} \tag{A.2}$$

where v is the Poisson ratio of the matrix, $\alpha_r = b_r/a_r$ is the aspect ratio of the rth spheroid, and q is given by

$$q = \frac{\alpha_r}{(1-\alpha_r^2)^{3/2}}[\cos^{-1}\alpha_r - \alpha_r(1-\alpha_r^2)^{1/2}], \tag{A.3}$$

when $\alpha_r \leqslant 1$.

From these results, we see that for spheres ($\alpha_r = 1$, $q = 2/3$),

$$S_{ijkl}^{(r)} = \frac{5v-1}{15(1-v)}\delta_{ij}\delta_{kl} + \frac{4-5v}{15(1-v)}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}). \tag{A.4}$$

If r refers to a typical flat compliant Hudson-crack (characterized by $\alpha_r \to 0$, $q \to 0$) then the only non-vanishing components are

$$S_{3333}^{(r)} = 1, \quad S_{3311}^{(r)} = S_{3322}^{(r)} = \frac{v}{1-v}, \quad S_{1313}^{(r)} = S_{2323}^{(r)} = \frac{1}{2}. \tag{A.5}$$

The above expressions for the $\mathbf{G}^{(r)}$ tensor for a spheroidal inclusion can also be used to evaluate the $\mathbf{G}_{\mathrm{d}}^{(rs)}$ tensor, provided that the aspect ratio $\alpha_{\mathrm{d}}^{(rs)}$ of the correlation function is taken to be identical with that of the associated inclusion (see also Ponte Castaneda and Willis, 1995).

If the spheroidal inclusions are embedded in a matrix material which is no longer isotropic then one needs to integrate a set of expressions over a finite range in order to evaluate the $\mathbf{G}^{(r)}$ tensor. In principle, the $\mathbf{G}^{(r)}$ tensor can be computed for inclusions having any shape, but then one looses the simplicity of Eshelby's ellipsoidal inclusions (e.g., Jakobsen et al., 2003a).

Appendix B. Evolution law for volume concentrations

If we denote by $\delta|\Omega^{(r)}|$ the (stress-induced) change in the volume $|\Omega^{(r)}|$ of the ellipsoidal domain $\Omega^{(r)}$ occupied by a single inclusion (or cavity) of type r, then it follows from the definition of the strain tensor $\epsilon_{kk}^{(r)}$ that

$$\frac{\delta|\Omega^{(r)}|}{|\Omega^{(r)}|} = \delta\epsilon_{kk}^{(r)}. \tag{B.1}$$

The results of Eshelby (1957), in conjunction with the concept of eigenstrain (e.g., Landau and Lifshitz, 1959), can be used to provide more justification to the above equation.

The total volume occupied by all inclusions is obviously given by

$$|\Omega| = \sum_{r=1}^{N} n^{(r)}|\Omega^{(r)}|, \tag{B.2}$$

where $n^{(r)}$ is the number of r-inclusions per unit volume. The above equation implies that

$$\delta|\Omega| = \sum_{r=1}^{N} n^{(r)}\delta|\Omega^{(r)}|. \tag{B.3}$$

From Eqs. (B.1) and (B.3), we get

$$\delta|\Omega| = \sum_{r=1}^{N} n^{(r)}|\Omega^{(r)}|\delta\epsilon_{kk}^{(r)}. \tag{B.4}$$

Thus,

$$\frac{\delta|\Omega|}{|\Omega|} = \sum_{r=1}^{N} v^{(r)}\delta\epsilon_{kk}^{(r)}. \tag{B.5}$$

where

$$v^{(r)} = \frac{n^{(r)}|\Omega^{(r)}|}{|\Omega|}, \tag{B.6}$$

as discussed (within the context of a statistical homogeneous system) by Jakobsen et al. (2003a).

It follows from the above equation that

$$\frac{\delta v^{(r)}}{v^{(r)}} = \frac{\delta|\Omega^{(r)}|}{|\Omega^{(r)}|} - \frac{\delta|\Omega|}{|\Omega|}, \tag{B.7}$$

where the last term on the right-hand side represents a correction factor taking into account the fact that the total volume of a representative volume element is also changing when the system is subjected to an applied effective stress.

Combining Eqs. (B.1), (B.5) and (B.7), we finally get

$$\frac{\delta v^{(r)}}{v^{(r)}} = \delta\epsilon_{kk}^{(r)} - \sum_{s=1}^{N} v^{(s)}\delta\epsilon_{kk}^{(s)}. \tag{B.8}$$

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